Characterizing Shifts in Students' Reasoning about Probabilistic Causality: A Microgenetic Study

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Abstract

The Next Generation Science Standards (Achieve, 2013) require middle school students to understand that "some cause and effect relationships in systems can only be described using probability." While research on causal induction suggests that summing across probabilistic instances is within children's causal reasoning repertoire (e.g., Gopnik, Glymour, Sobel, Schulz, Kushnir, & Danks, 2004), evidence also suggests that learners hold a preference for deterministic causation (i.e., cause reliably leads to effect) (e.g., Schulz & Sommerville, 2006). However, in certain circumstances, students demonstrate the ability and tendency toward probabilistic reasoning (i.e., cause and effect relationship is unreliable) (Grotzer, Solis, Tutwiler, & Powell, in review). Exploring these instances of probabilistic reasoning in detail can inform curricular interventions to help students learn about complex probabilistic phenomena in science. Adapting Siegler's (1995) microgenetic taxonomy of dimensions of cognitive change, the present paper characterizes the path, breadth, sources, rate, and variability of change in students' reasoning about stochastic causal phenomena over several sessions.

Theoretical Framework

Early research found that children expected cause-effect relationships to be reliable and treated one-to-one correspondence between causes and effects as a key feature of causality (e.g., Bullock, 1985; Shultz, 1982). Researchers argued that determinism was one of a set of fundamental principles that learners applied in their causal reasoning (e.g., Bullock, Gelman, & Baillargeon, 1982). However, more recent research suggests that even preschoolers can follow Bayesian rules to sum across experiences in their causal judgments. Gopnik and colleagues (2004) argue that young children override imperfect correlation and are able to use different patterns of probability in contiguity to make accurate causal inferences (Kushnir & Gopnik, 2007).

While the work by Gopnik and colleagues suggests that children implicitly sum across probabilistic instances in discerning causality, other work suggests that children explicitly reject probabilistic causality in some instances, preferring deterministic causal explanations when considering machine-like mechanisms (Schulz & Sommerville, 2006), biological phenomena such as illness (Kalish, 1998), and patterns of effects that occur over repeated instances in a distribution (Metz, 1998). Our own work has shown that students demonstrate deterministic reasoning when asked to respond to and explain stochastic scenarios (Grotzer et al., in review). Whether playing a board game, discussing the outcomes of planting seeds, or explaining the workings of a gumball machine, students in kindergarten through sixth grade demonstrated a preference for deterministic explanations of causal relationships, expecting that events would be preceded by corresponding causes. That said, the preference for deterministic reasoning seemed more or less prominent depending on the domain of the causal phenomenon. For example, social scenarios where students were asked whether an individual could reliably predict how someone else would react in a social exchange tended to elicit probabilistic explanations that referred to an observer's inability to know how someone else will respond in a social interaction. By contrast, extant literature shows that children appear to prefer deterministic explanations for physical mechanisms (Schulz & Sommerville, 2006) but also to extend this to other domains such as germs (Kalish, 1998). One possible explanation for differences in the way learners respond to probabilistic instances in different domains is their prior knowledge and personal experience; that is, familiarity with the task domain and children's mechanism knowledge may inform their causal explanations. For example, Kuzmak and Gelman (1986) found that children could express some knowledge of the difference between deterministic and random outcomes when they could see how the mechanism operated. Mechanism knowledge has played a strong role in children's reasoning in other instances in which they could not rely on co-variation alone (e.g., Sandoval & Cam 2011) and researchers have argued that mechanism is a key component in causal reasoning (e.g., Ahn & Kalish, 2000).

A final observation in our previous work was that students' reasoning exhibited shifts within and across tasks from deterministic to probabilistic explanations, especially when responding within supported scenarios. These findings suggest that research into students' reasoning about stochastic phenomena may inform the design of curricular and instructional interventions that support explicit reasoning about probabilistic causal relationships in science. In the present qualitative study we analyzed data from Grotzer et al. (in review) to better understand the nature of change in students' reasoning according to the dimensions set forth by Siegler (1995). We asked: how can the path, rate, breadth, sources, and variation of students' shifts from deterministic to probabilistic reasoning be characterized within and across study sessions?

Methods

A microgenetic study, consisting of multiple observations and intensive analyses of a small number of students over time was conducted to surface patterns of change in students' reasoning and explanations (Calais, 2008; Siegler & Crowley, 1991).

Sample and Setting. Participants included 16 students (4 kindergarteners, 4 second graders, 4 fourth graders and 4 sixth graders) over the course of a school year. Students attended public schools in an urban district, with a predominantly Latino and Black population and diverse SES.

Procedures. Students participated in semi-structured interviews during tasks in four domains (games, biological, mechanical, and social) that varied in their level of stochastic behavior (see Table 1 for a summary of the tasks). As students participated in a task (e.g., playing a game, planting seeds, watching videos of a social interaction), researchers asked them to explain their expectations and reasoning about the outcome in the causal relationship being presented (e.g., a move in the game, seeds growing, or the response of one individual to another's behavior). In latter sessions, scaffolds that made use of familiar examples and compared analogous causal forms in different problem contexts were incorporated in the form of design studies (Brown, 1992; Collins, 1999). At least four sessions were conducted with each student, but in keeping with the microgenetic study design (e.g. Metz 1985; Siegler and Crowley

1991), the number of sessions was variable to follow students' reasoning as it changed throughout the study. A total of 108 sessions (minimum of 4, maximum of 10, mode of 7 per student; 30-45 minutes in length) were conducted. Sessions were audiotaped, videotaped, and transcribed for later coding and analysis.

[Insert Table 1 here.]

Data Sources and Analysis. Session transcripts were analyzed using Atlas.ti to identify instances when students revealed deterministic reasoning (i.e., cause always leads to an immediate effect) and when they revealed probabilistic reasoning (i.e., relationship between cause and effect is uncertain). Transcripts were independently coded by two researchers with 25% overlap to assess reliability. Kappa analyses demonstrated strong interrater reliability (above .80 for each grade). Overall findings based on this initial coding are reported elsewhere (Grotzer et al., in review). Following the initial coding, memos were developed for individuals to document how their reasoning changed over time. Transcripts and memos were analyzed using an adapted version of Siegler's (1995) microgenetic taxonomy of dimensions of cognitive change to characterize the path, breadth, sources, rate, and variability of change in students' reasoning.

- *Path of change:* the sequence of "qualitatively distinct understandings" through which children progress "on their way to mature competence" (p. 228).
- *Breadth of change:* how narrowly or broadly children generalize new conceptions they gain.
- *Sources of change:* the "experiences [and interventions] that might contribute to cognitive change" (p. 233).
- *Rate of change:* the speed with which children progress from initial conceptions through new conceptions toward mature conceptions.
- *Variability of change:* differences "among individual children's change patterns" (p. 233).

Results

All students demonstrated some instances of probabilistic reasoning, with seven students showing shifts in reasoning over time. Shifts were determined when students' responses turned from mostly deterministic to mostly probabilistic (Grotzer et al., in review). As discussed below, however, students' probabilistic reasoning emerged in different ways. Here we describe the nature of students' probabilistic reasoning, utilizing students' responses to illustrate the themes regarding the path, breadth, sources, rate, and variability of change in their reasoning.

Path of Change

Two predominant paths for change were apparent in the data: a shift from deterministic towards more probabilistic responses and a shift to a more balanced perspective between the two reasoning patterns.

From deterministic towards more probabilistic. Four students demonstrated a pronounced shift from deterministic to probabilistic reasoning. Elias[1], a fourth grader, started out with 60% to 100% of his responses expressing deterministic reasoning, but in latter sessions expressed up to 70% probabilistic responses. For example, he initially believed that there was a pattern in the way a board game behaved; although he did acknowledge that he might not be able to predict where a spot in the game would open up, he indicated the deterministic belief that an order existed to how the game behaved. As Elias continued to consider his own inability to figure out the workings of the games he was playing, he expressed stochastic explanations: "Maybe it just goes in random order." By the end of the study when the task was to make connections across tasks and make analogies to real-world scenarios, most of Elias' responses were probabilistic. Other students similarly exhibited a clear and stable change that indicated their ability to notice and explain probabilistic phenomena.

*From deterministic to deterministic/probabilistic***.** Three students started out with deterministic reasoning and as sessions progressed, became less deterministic, resulting in some sessions with mostly deterministic responses and other sessions with mostly probabilistic responses. The shift toward probabilistic reasoning was not as pronounced or stable. Rajon, a second grader, started the study giving as many as 80% to 100% deterministic responses in the early sessions. Halfway through the sessions, his responses were closer to 50% deterministic in some cases, and he used "random" as a term to describe the nature of causal relationships. What is important to highlight here is that although this was still considered a shift because responses moved from mostly deterministic to mostly probabilistic in some sessions, students were still entertaining both types of causal patterns across sessions as they formulated their explanations.

A third pattern in students' reasoning was detected. Some students were probabilistic at the outset and maintained that stance throughout the study; specifically, five students expressed probabilistic reasoning early on in sessions and maintained this type of reasoning throughout subsequent sessions. Kendra, a second grader, began the study giving 42% deterministic/57% probabilistic responses and had sessions where 100% of her responses were probabilistic later in the study. In her first session, she described the card game she was playing in clearly probabilistic terms ("sometimes"),

> *K: Sometimes when you press this nothing happens, and then when you press it again something might happen or something doesn't and sometimes when it pops up it stays in there [the cards that come half out] and sometimes it just falls on the table. I don't get that…and sometimes it just goes "boing" [propels out of her chair] like a rabbit hopping everywhere.*

At the end of the study, she summarized her understanding by explaining that "sometimes" she could "predict" the outcome of an event and "sometimes" she could not.

Breadth of Change

Near transfer. Some students were able to articulate probabilistic reasoning, but only within the task at hand or the domain of the task being presented. When asked to produce their

[1] Pseudonyms are used throughout.

own examples or compare to other tasks, their reasoning was deterministic again. Although Andre, a sixth grader, was able to reach a probabilistic understanding in a previous task (when making predictions about how many M&Ms would come out of a gumball machine), he did not transfer this understanding to his interpretation of a social scenario, when he expressed that the protagonist would be able to predict another's response 100% of the time.

R: OK. Um, so we had talked before about whether or not there's a way to predict how many M&Ms are going to come out. So, can you remind me a little bit about what your thoughts were on that?
A: Well, again, you can't be 100% sure -- how much is going to come out. That's why it's called an estimate. That's when you can't be 100% sure. You can probably be 99.9% sure, 50% sure, any percent sure, but not be always, 100% sure.
R: OK. Do you think that the brother can be 100% sure that his sister was going to get mad when he took her markers?
A: Yes.
R: Why is that?
A: Because every time he took it, the girl would always yell at him.
…
R: OK. So, do you think that every, as long as she was there using the markers and saw him take them, she'd get mad?
A: Yes.

<u>*Far transfer*</u>**.** By contrast, other students were able to provide real-world examples and connect to other tasks they had experienced in the study. Elias made the following connections across the tasks in the study:

R: What do you think is similar in all these games and activities?
E: The thing that's similar about them is that… they always, you never know what's going to happen. For example, for the video, she doesn't know when her mom is going to come or when she's going to notice that he's taking the markers. And, also, with the games, you never know when the bear is going to wake up or if the carrot is going to make a hole appear. The incubator… you don't know if the eggs are going to hatch in a certain amount of days… and with the M&M machine, you don't know how many M&Ms are going to come out in one turn. And, also with the planting, you don't know how long it takes for the plant to grow.

He also made connections to real-world scenarios,

R: How is [a volcano] similar to everything that we've been talking about?
E: Because you don't know when it's about to erupt, and you don't know…um, when the bear is going to wake up or when the hole is going to come, or how many M&Ms…[unclear].

Sources of Change

As discussed, an important factor influencing students' ability to reason about probabilistic phenomena was the domain of the scenario they were presented or task they were completing (Grotzer et al., in review). However, when examining students' thinking closely, other factors seemed to arise as possible sources of change in students' reasoning.

Subjective uncertainty. We observed some students moving from a deterministic stance to exhibiting subjective uncertainty during the course of a session and then expressing probabilistic reasoning as sessions progressed. The concept of subjective uncertainty refers to instances when students believed there was a deterministic outcome or causal pattern in the situations they observed but that they could not decipher the pattern themselves. For example, although Khloe, a kindergartener, believed that there was a pattern in the way a board game behaved, she did not believe she could predict where a spot in the game would open up:

R: Do you know where the hole is going to open next?
K: No.
R: Is there a way to know?
K: Maybe if you turn around.
R: If you turn the game around?
K: Yeah. Like this…[K makes a lifting motion]…maybe then.

Khloe's response demonstrated that she believed understanding the mechanism of the game would allow one to make deterministic predictions about the outcome of certain moves, but this mechanism was not accessible (or knowable) to her at the time.

This expression of subjective uncertainty often occurred as a result of making predictions that did not turn out to be correct or realizing over time that the pattern of results they expected was not exactly demonstrated in the evidence they observed in a game or given scenario. For example, when students played games, they received immediate feedback about whether their predictions were correct or not and seemed to keep track of how well their predictions turned out. Students referred to this feedback to draw conclusions about the causal structure of events. Kendra referred to the observed evidence of a card game to inform her probabilistic reasoning:

R: So do you think you can know how many cards are going to come out?
K: NO! Because once it came out 2 cards, then it came out 5 cards, then it came out 4 or something like that.

We observed students who were able to move from "I don't know" (subjective uncertainty) to "It's not knowable" (objective uncertainty). Realizing that it was not knowable often accompanied the concept that the cause to effect ratio was not one to one and that complexity and/or randomness entered into the equation. Subjective uncertainty appeared to open the way for students to move toward probabilistic thinking and maintain this reasoning in subsequent sessions.

<u>Prior experience</u>. When students did not have access to direct evidence, they referred to prior experiences and knowledge to reason about the tasks and scenarios they were presented with. Iris, a fourth grader, talked about her personal experiences to support her probabilistic reasoning about hatching eggs:

I: Or maybe there'll be seven.
R: Oh. Why do you think?
I: Because when I watched the movie of Charlotte's Web, it had, like, a rotten egg.
R: Yeah. And what does that tell you? What are you figuring out from that?
I: That not all chicks hatch.

<u>Scaffolds</u>. The fact that students referred to their personal experience is perhaps not surprising given that familiar scenarios were chosen for this study to elicit students' existing knowledge. Additional instructional moves were included to scaffold students' reasoning. We asked students to draw analogies to real-life events and to compare across scenarios and sessions that had been presented in the study. Elena, a sixth grader, referred to her understanding of different tasks she was presented to explain the stochastic evidence she observed across sessions:

E: In Don't Wake Up the Bear, that game, when you click the button, you never know how many times it's going to get up. And then Uno when you click the button, you never know how many times you gonna click it before it come out. When you plant seeds, you don't know how much is going to grow.

As students drew cross-domain connections, some of them saw the similarities in their reasoning that helped them to solidify working understandings of the stochastic evidence. In some cases, these scaffolds helped them to broaden the scope of their understanding to other examples that they constructed themselves.

Rate of Change

The rate of change seemed to coincide with the patterns we found in the path of children's reasoning. The four students who demonstrated a pronounced shift from deterministic to probabilistic tended to exhibit this shift in the latter two or three sessions of the study, when they were beginning to make (prompted or unprompted) connections to previous tasks or life experiences. The three students who went from mostly deterministic in early sessions to a mixture of deterministic and probabilistic reasoning, tended to exhibit this shift halfway through their sessions. For example, Ruby, a fourth grader, gave 75% probabilistic responses in her fourth session, then had two sessions where she gave mostly deterministic responses, and then returned to giving mostly probabilistic responses in the last session. As expected, students who exhibited probabilistic reasoning at the outset did so in the first session and throughout the study.

In this study, however, it is difficult to assess the rate of change because the sequence of tasks was confounded with the type of task. Therefore, it is possible that some tasks did more to elicit probabilistic causal reasoning and this influenced the rate at which students shifted in their reasoning. As discussed in the methods section, some students had multiple sessions with one task because they were reasoning actively about that task and in the spirit of microgenetic, design-based research, additional sessions were added to discern how students' reasoning was impacted.

Persistence in pattern-seeking actually slowed the progress of some students but might also have resulted in a deeper sense of what leads to probabilistic causal patterns. An example of this is Rajon. In the first session with the Funny Bunny board game, he discovered that the hole that opened up moved around the hill that is the game board. He immediately started looking for a pattern and relating the game to another game that he had played. When that did not help him figure out what was going on, he looked for other evidence and started to focus on what moved on the game board. However, even by the fifth session, he still struggled with the complexity of what explained where the hole on the game board would open. At that point, he shifted from a deterministic to a probabilistic explanation. He questioned the existence of a deterministic and knowable pattern and instead of saying he did not know (subjective uncertainty), he started to say that, "you can't know." These shifts in how he reasoned about the game were accompanied by a "letting go" of discerning the pattern. He started to talk about luck and when a rabbit fell through a hole, he exclaimed:

R: Lucky! I'm lucky!

When asked what he thought about this, he replied:

R: You just can't know.

Thus, while some students like Rajon exhibited a slow shift over several sessions, it seemed that exploration of phenomena over time was productive in helping students come up with probabilistic explanations, even if initially this was not reflected in the number of deterministic vs. probabilistic responses.

Variability of Change

As the previous sections demonstrate, children's changes in reasoning were variable. Individual student narratives illuminate micro-shifts that gave way to more sustained change in reasoning; they also show that even when students were capable of understanding probabilistic phenomena, their thinking may have reverted to deterministic reasoning in unfamiliar domains or when the evidence seemed to support a deterministic stance. Figure 1 presents the trends of probabilistic reasoning across a subset of sessions for one student per grade. A line representing the average trend across all 16 students highlights the variability within and across students. Kindergartener Tanika's line represents the most dramatic growth towards probabilistic causal reasoning while sixth grader Elena's reveals an ability to detect probabilistic causal features from the outset, although the strength of probabilistic reasoning increased or decreased depending on the session.

[Insert Figure 1 here.]

Discussion

The present findings demonstrate texture in students' responses over time. These results suggest a few things. First, students' grasp of a complex causal pattern like probabilistic causality may not exhibit a smooth learning trajectory. Some students will start out thinking about phenomena in stochastic ways, others may reason stochastically over time, and yet others may entertain both probabilistic and deterministic ideas. Even when students do exhibit probabilistic thinking, this reasoning may be limited to a domain or subset of phenomena and may not transfer to novel circumstances. Second, students may be prompted to think stochastically about phenomena because of evidence they observe, prior knowledge, or instructional scaffolds. Furthermore, stable change may involve an incubation period and opportunities to solidify understanding that has been developing over time and across cases. It may be up to educators to draw on a variety of sources to help students think about complex stochastic phenomena. Finally, because of the variability inherent in learning, science instruction at all grade levels requires a flexible design that adapts and responds to the shifts in reasoning, strengths, and interests of students. Understanding the variability in the path, breadth, rate, and sources of change in students' reasoning should help science educators and researchers in developing learning opportunities that honor the nuanced and rich learning trajectories of individual students while supporting all students in dealing with stochastic causality and statistical data as a form of scientific evidence.

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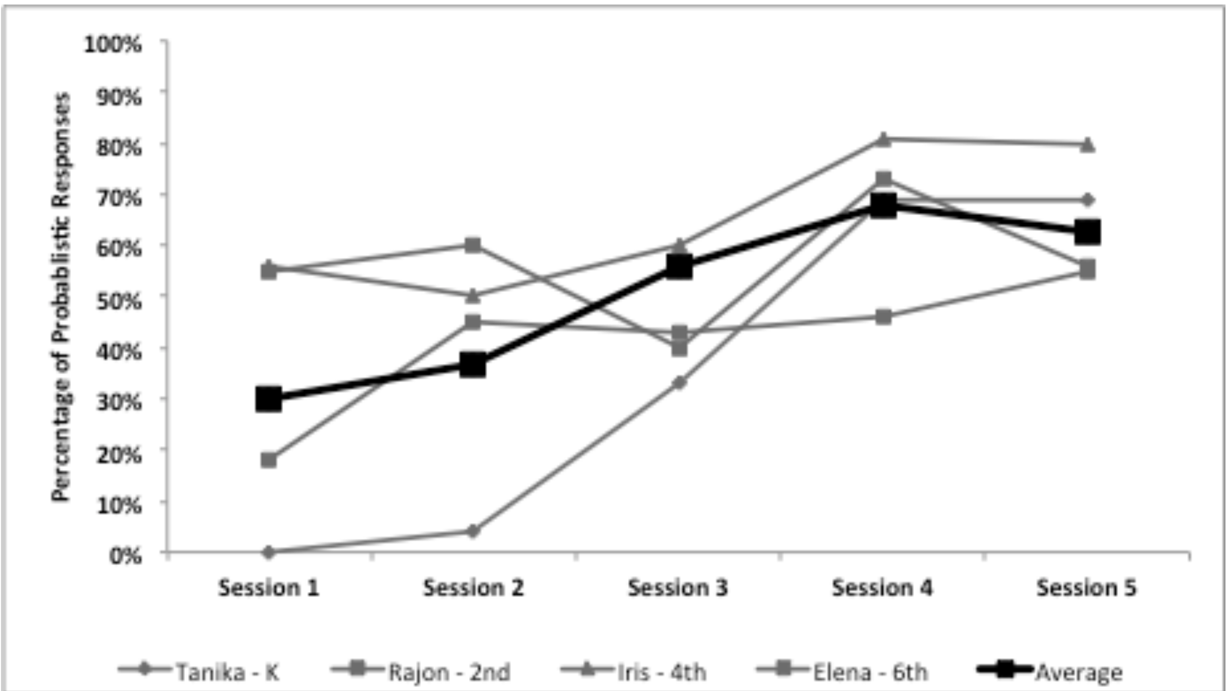
Table 1. Summary of Tasks and Requirements for Reasoning About Probabilistic Causality in Each

Content Domains and Tasks
Games
Funny Bunny (commercial game by Ravensburger). The goal is to be the first one to move your rabbits along a path with two loops up the hill to the top of a big carrot. Cards tell how many steps to move, however, some cards direct players to click the carrot in the middle of the game board causing a hole to open up somewhere along the path most of the time and one's rabbit can fall through. The location of where the hole opens up gives the appearance of being stochastic in the following ways. Initially, there is no indication that the hole in the path moves. Upon the turn of the carrot, the hole moves along the path, alternating between the top row of the path and the bottom row of the path. Periodically, no hole opens at all. The cognitive load of figuring out which hole will open involves detecting that some spaces hold the possibility of opening (nine out of 26 are "wiggly" or "soft" whereas others never open and are always safe); detecting that the holes move in a clockwise fashion around the board; that they alternate between the top and bottom rows; and that the hole also disappears at a certain point in the rotation. **Last Bunny Standing** (modified version of Funny Bunny). The child has to figure out where to put a bunny on each turn so that it will be safe when the carrot is clicked. It eliminates some cognitive load related to game strategy (how many rabbits to put on the board and how the randomness of the shuffled cards interacts with outcome) and focuses directly on the goal of figuring out where the hole opens given its seemingly stochastic nature. Turning the game over would reveal the mechanism and offer information that one could use to deduce a pattern. However, it involves transferring that information dynamically to the top of the board and being able to track how two moving plates under the game interact to result in whether and where a hole opens. **Don't Wake the Sleeping Bear** (modified from a Hasbro game entitled, "Don't Wake Daddy" for use with subjects from single parent homes and racially diverse schools.) The goal is to get to the finish line without waking up the sleeping bear. However, when spaces are landed upon, the player must push the button on an alarm clock a given number of times and if the bear pops up, must return to start. The number of alarm clock pushes that caused the bear to pop up ranged from 6 to 20 and each of the three games that were used had a different pattern of when the bear would pop up. However, if the students were not tracking how many pushes others had entered, it could pop up on the first push (presuming five pushes occurred during other turns). There is no visible mechanism to account for what happens. **Uno Attack** (a variation of the game, Uno, where the player attempts to be the first to get rid of all of his or her cards, this version has an automated card dispenser). A player pushes the button on the dispenser and sometimes it dispenses cards (a seemingly random number of them) though most times it does not. There is no discernible regular pattern. There is no visible mechanism to account for what happens. When the dispenser is opened up to add cards, one can see a flywheel, however, it does not work when opened up so it is not possible to test under what conditions it shoots cards or not.
Biology
Seed Planting: Students were told that they needed to have a certain number of bean plants to give to specific people in a few weeks. They were then given a peat pot, soil, and seeds and invited to plant the

number of seeds that they thought they should plant in order to end up with necessary plants. They engaged in the task two to three times. The researchers manipulated the number of seeds and growth patterns to present different outcomes the following week as a basis to interview students' interpretations of different levels of stochastic results. **Hatching Eggs:** Students were asked to predict what the inside of an incubator might look like in 22 days after eggs were set inside it. They were told that eggs typically hatch in 21 days. They were given a drawing showing eight eggs and were given an opportunity to draw the outcome later. Afterwards, they were probed on what causes the eggs to hatch, their experiences hatching eggs, and whether they had ever seen an outcome where less than the number of eggs hatched.
Social
Social Response Patterns: Students were shown two brief video clips. In one, a girl is calling her mom for help with her homework. The rate of calling to response varies as follows: 1) girl calls, mom responds, 2) girl calls, calls again, calls again, mom responds; 3) girl calls, calls again, calls again, calls again, and calls again, then mom responds. Students were asked what causes the mom to come and how the versions are different from one another. A second video shows a boy pestering his sister by taking her markers and she responds. The rate of pestering to response varies as follows: 1) boy takes marker, sister responds, 2) boy takes a marker, takes another marker, takes another marker, sister responds; 3) boy takes a marker, takes another marker, takes another marker, and another, and another marker, then the sister responds. **Cheating and Getting Caught:** Students were asked "If someone cheats on a test or homework, do they always get caught?" "Can you think of ways that cheating and getting caught are like the way that the game worked?
Mechanical
Candy Dispenser: Subjects were shown a candy dispenser and were given coins and invited to make it work. The dispenser dispensed between zero and five candies with each turn with a mode of five. The actual mechanism for dispensing candies was not visible given the number of candies in the dispenser. Subjects could detect some information about the mechanism, however, because the handle was less easy to turn on some turns when it would dispense no candies and on others turning very slowly appeared to yield higher returns.
Across Domain Connection-Making Scaffolds
Connections: Students were given probabilistic examples, some of which were related to earlier study tasks and some that were not but seemed likely to be within their experience, to reason about and were asked to think about other examples that might be like them. **Analogies:** Using "mutual alignment" (e.g. Kurtz, Miao, & Gentner, 2001), students were asked to map back and forth between analogical problems making attempts to discern similarities and differences and to use each to further inform understanding of both.

*Reprinted with permission from (Grotzer, Tutwiler, Solis, & Duhaylongsod 2011).

Figure 1. Trends of Probabilistic Responses (Percentage) across Five Sessions for One Student per Grade



*Average line represents trend across all 16 students in the study.